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Unit 2

Section 2.1 - Rates of Change and Limits

$$\text{Average Speed} = \frac{\text{Distance covered}}{\text{Elapsed time}}$$

Example

A rock breaks loose from the top of a tall cliff. What is its average speed during the first 2 seconds of fall?

Recall that $y = 16t^2$

Definition of a Limit

Gives us a language for describing how the outputs of a function behave as the inputs approach some particular value

some limit (#)

$$\lim_{x \to c} f(x) = L$$

Read as "The limit of f of x as x approaches c equals L"

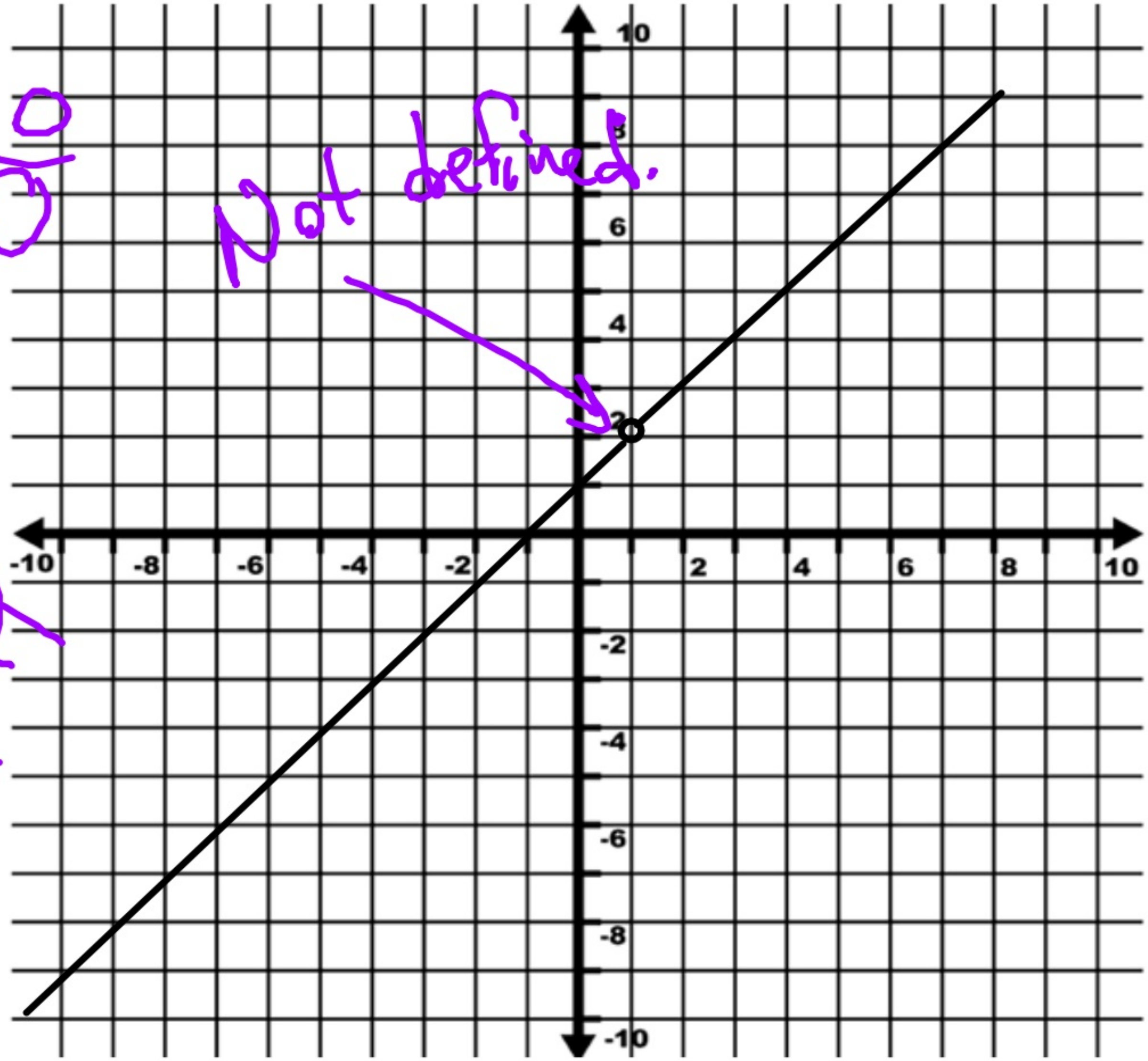
Examples

$$f(x) = \frac{x^2-1}{x-1} = \frac{0}{0}$$

$$\lim_{x \to 1} f(x) = ?$$

$$\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

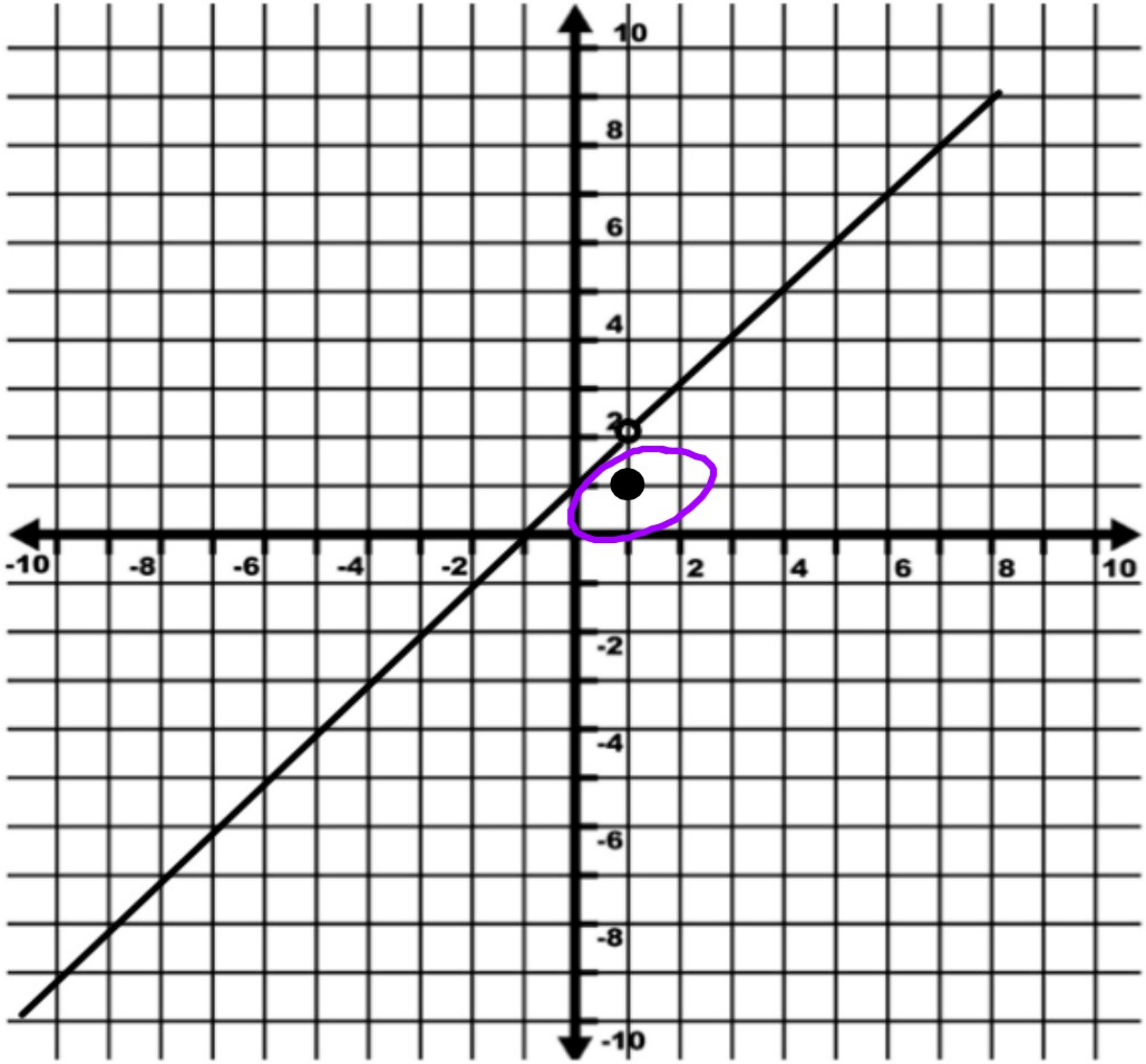
$$x+1$$



$$(x) = \frac{x^2-1}{x-1}\ ,\ x \neq 1$$

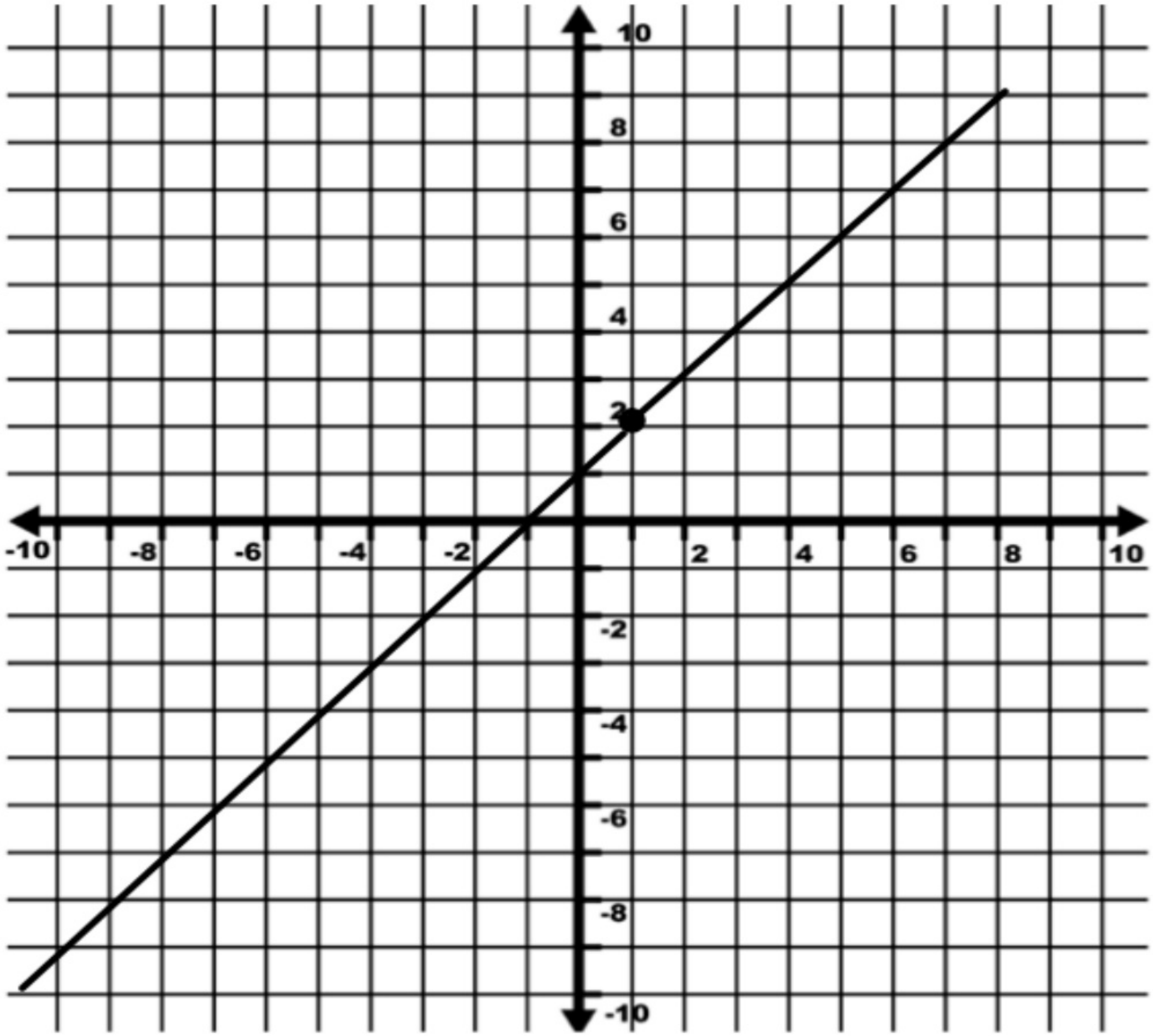
$$= 1\ ,\ x = 1$$

$$\lim_{x \to 1} g(x) = ?$$



$h(x) = x + 1$

$\lim_{x \to 1} h(x) = ?$



Two Sided Limits

$$\lim_{x \to \#} f(x) = L$$

$$\lim_{x \to \#^-} f(x)$$

One Sided Limits

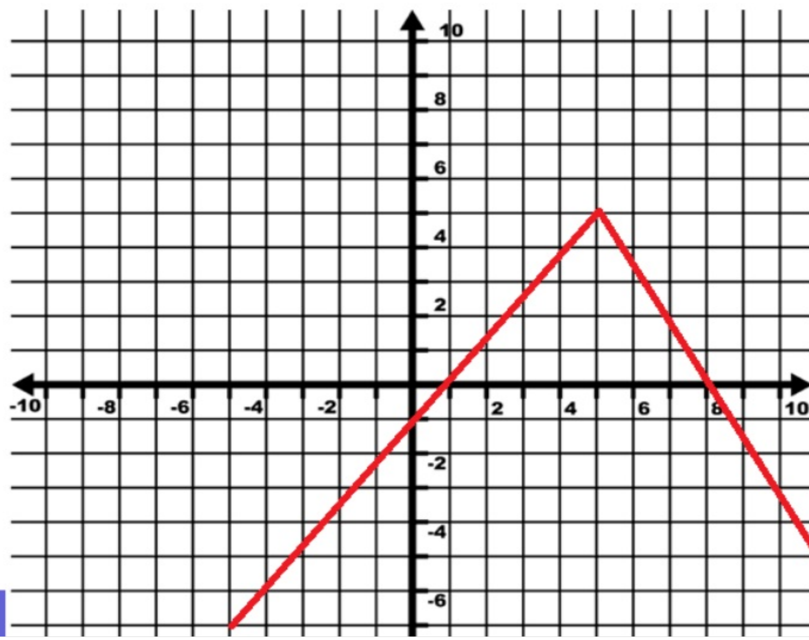
from left hand side

$$\lim_{x \to \#^-} f(x) = L$$

from right hand side

$$\lim_{x \to \#^+} f(x) = L$$

$x \to \#^+$



In order for a Limit to exist,

the one sided limits MUST be equal.

If not, we say the Limit

DOES NOT EXIST or D.N.E.

Properties of Limits

If L, M, c, and k are real numbers, and

$\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then

1.) Sum Rule : $\lim_{x \to c} (f(x) + g(x)) = L + M$

The limit of the sum of two functions is the sum of their limits

2.) Difference Rule : $\lim_{x \to c} (f(x) - g(x)) = L - M$

The limit of the difference of two functions is the difference of their limits

3.) Product Rule : $\lim_{x \to c} (f(x) * g(x)) = L * M$

The limit of a product of two functions is the product of their limits

4.) Constant Multiple Rule : $\lim_{x \to c} (k * f(x) = k * L$

The limit of a constant times a function is the constant times the limit of the function

5.) Quotient Rule : $\lim_{x \to c} (f(x) / g(x)) = L / M$, $M \neq 0$

The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero

6.) Power Rule : $\lim_{x \to c} (f(x))^{r/s} = L^{r/s}$

The limit of a rational power of a function is that power of the limit of the function, provided the latter is a real number

Example

$$\lim_{x \to c} (x^3 + 4x^2 - 3)$$

$$\lim_{x \to c} x^3 + \lim_{x \to c} 4x^2 - \lim_{x \to c} 3$$

$$c^3 + 4c^2 - 3$$

Example

$$\lim_{x \to c} \frac{x^4 + x^2 - 1}{x^2 + 5}$$

$$\frac{x+6}{x+1} \neq 6$$

Polynomial and Rational Functions

$f(x) = 3x^4 + 2x^2 - 5$

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ is any polynomial function and c is any real number, then

$$\lim_{x \to c} f(x) = f(c) = a_n c^n + a_{n-1} c^{n-1} + \dots + a_0$$

$f(c) = 3c^4 + 2c^2 - 5$

a.k.a. - You can substitute!!!

Example

$$\lim_{x \to 3} [x^2 (2 - x)] =$$

Example

$$\lim_{x \to 2} \frac{x^2 + 2x + 4}{x+2}$$

Example

$$\lim_{x \to 0} \frac{\tan x}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{\sin x}{\cos x} \cdot \frac{1}{x}}{x \cdot \frac{1}{x}}$$

$$\lim_{x \to 0} \frac{\sin x}{x \cos x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

Example

$$\lim_{x \to 2} \frac{x^3 - 1}{x - 2} = \text{DNE}$$

$$\lim_{x \to 2^+}$$

$$\lim_{x \to 2^-}$$

Find limits analytically

1.) $\lim_{x \to 2} x^2 - 3x + 4 = 2$

2.) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x \to 1} (x+1) = 2$

Example

$$\lim_{x\to 1} \frac{x^3 - 1}{x - 1}$$

$$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

$$\lim_{x\to 1} (x^2+x+1) = 3$$

$$\begin{array}{r}
x^2 + x + 1 \\
x-1 \enclose{longdiv}{x^3 + 0x^2 + 0x - 1} \\
-x^3 + x^2 \qquad\qquad \\
\hline
x^2 + 0x - 1 \\
-x^2 + x \qquad \\
\hline
x - 1 \\
-x + 1 \\
\hline
0
\end{array}$$

$$\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0} \frac{x\left(\sqrt{x+1}+1\right)}{x+1+\sqrt{x+1}-\sqrt{x+1}-1}$$

$$\lim_{x\to 0} \frac{\cancel{x}\left(\sqrt{x+1}+1\right)}{\cancel{x}}$$

$$\lim_{x\to 0} \left(\sqrt{x+1}+1\right) = 2$$

$$\lim_{x \to 5} \frac{x-5}{x}$$

$$\lim_{x \to 0} \frac{x}{|x|} = \text{DNE}$$

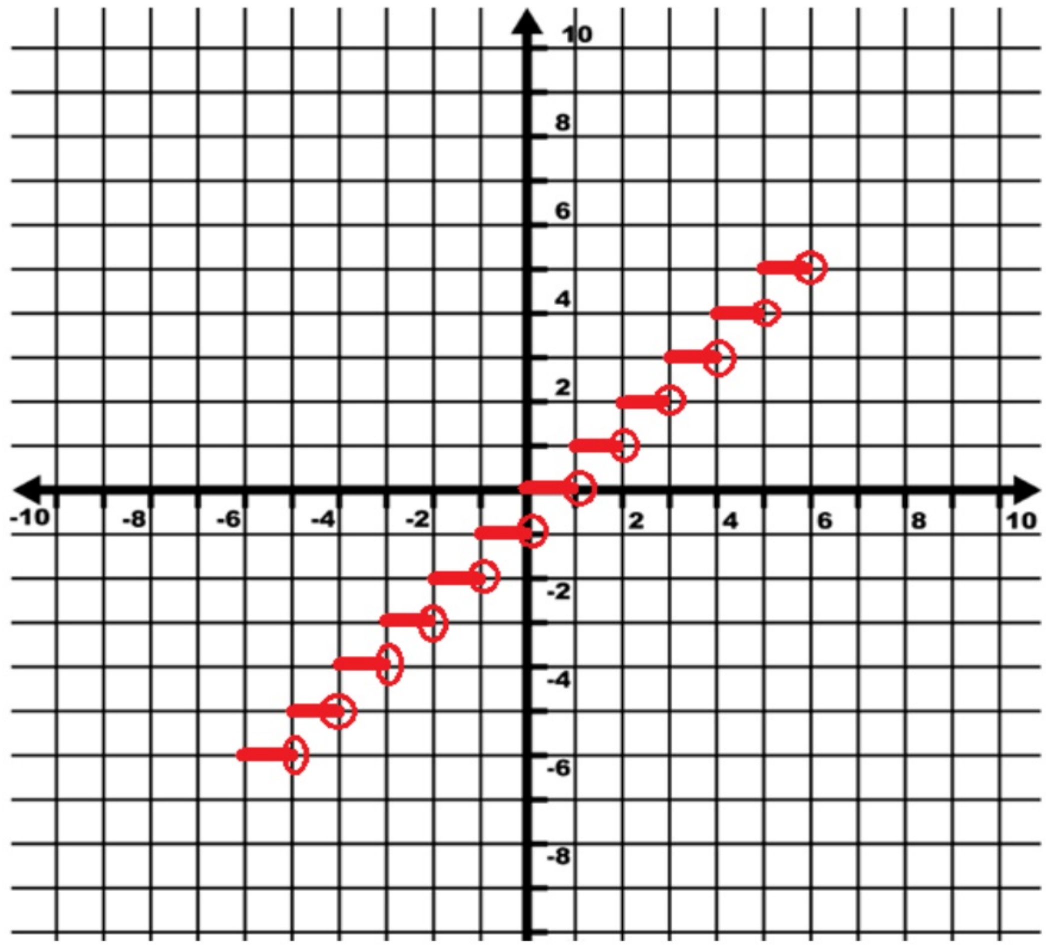
$$\lim_{x \to 0^+} \frac{x}{x} = 1$$

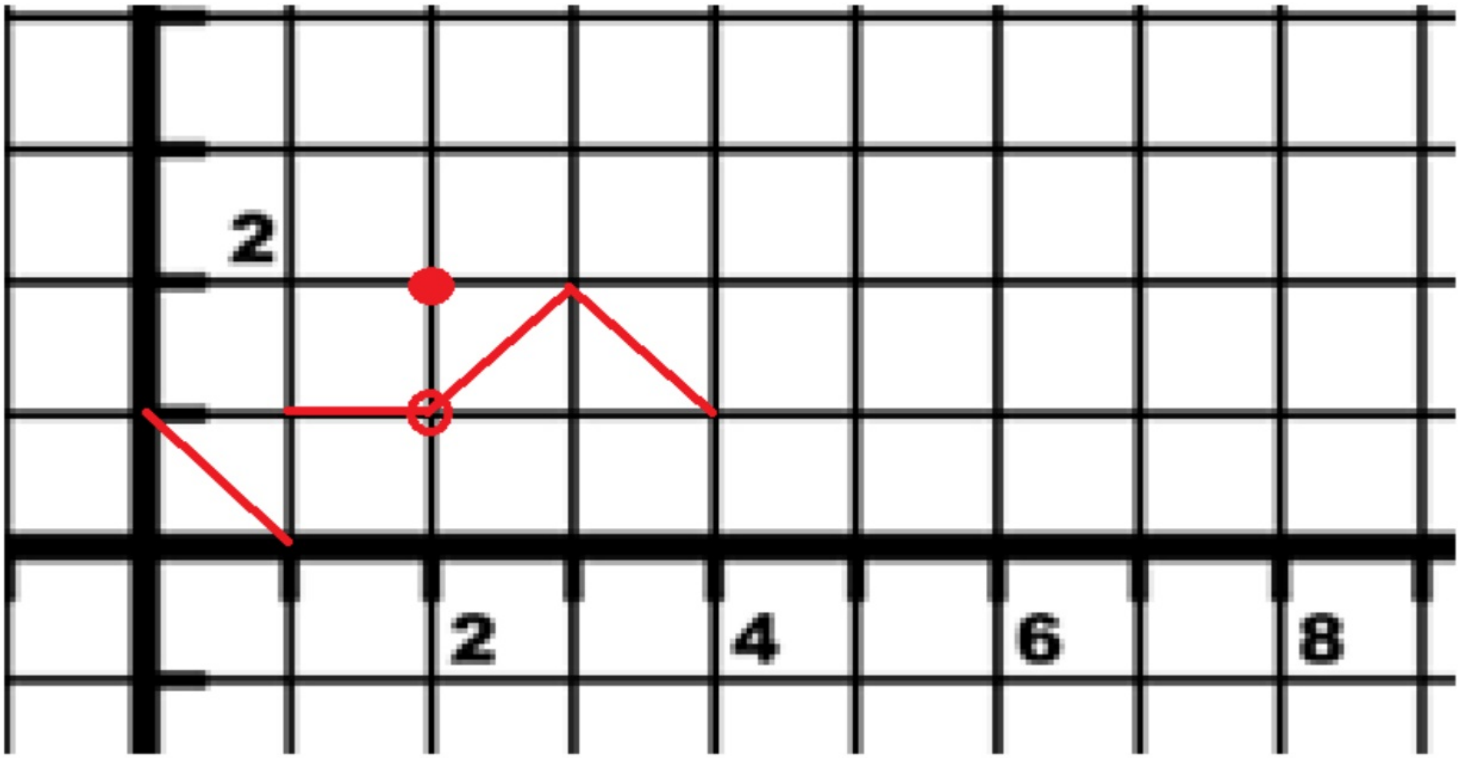
$$\lim_{x \to 0^-} \frac{x}{-x} = -1$$

$$\lim_{x \to 3^+} \text{int } x =$$

$$\lim_{x \to 3^-} \text{int } x =$$

y = int x





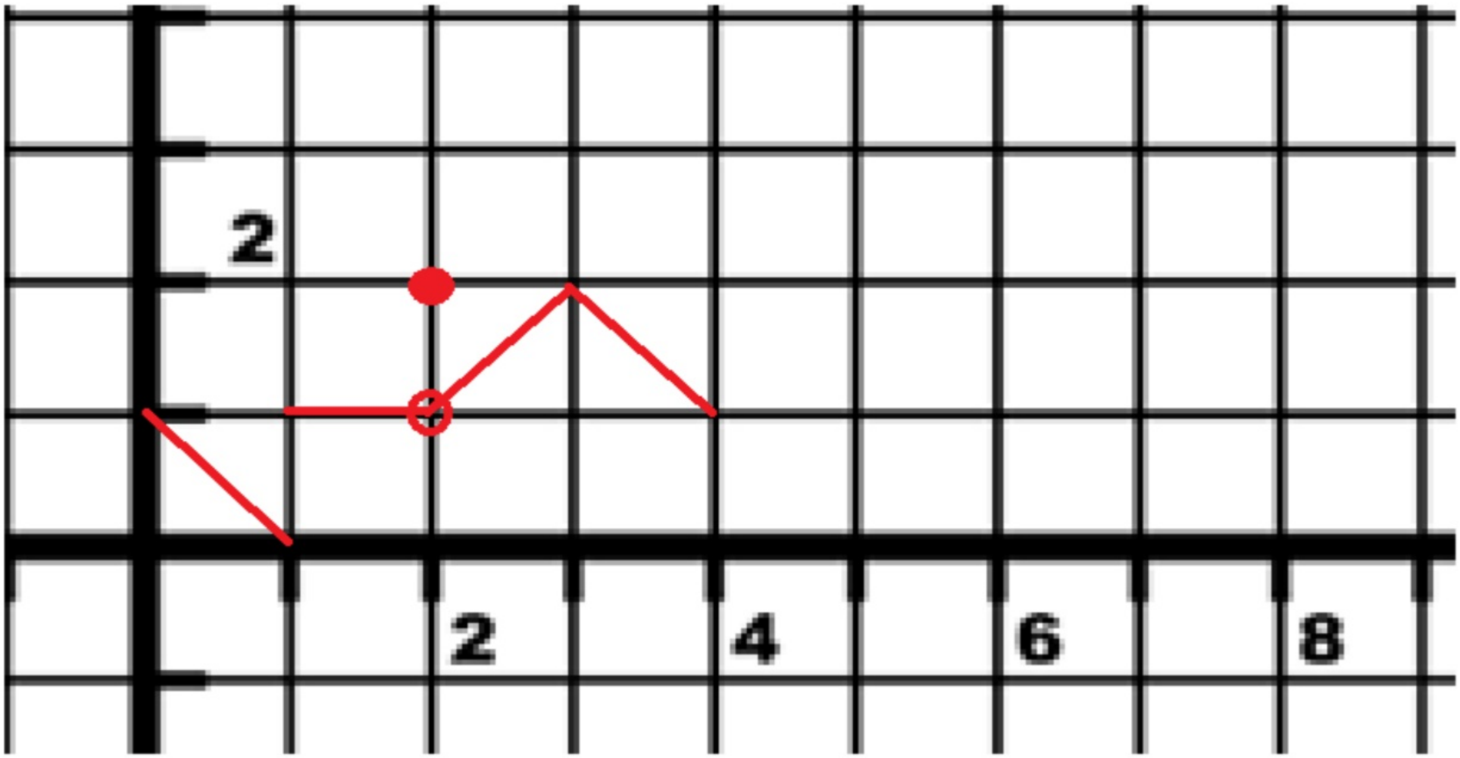
$$f(x) = \begin{cases} -x+1, & 0 \leqslant x < 1 \\ 1, & 1 \leqslant x < 2 \\ 2, & x = 2 \\ x - 1 & 2 < x \leqslant 3 \\ -x + 5, & 3 < x \leqslant 4 \end{cases}$$

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 1^-} f(x) =$

$\lim_{x \to 1^+} f(x) =$

$\lim_{x \to 1} f(x) =$



$$\lim_{x \to 2^-} f(x) =$$

$$\lim_{x \to 2^+} f(x) =$$

$$\lim_{x \to 2} f(x) =$$

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x \to 0} \frac{\sin 4x}{4x}$$

$$\lim_{x \to 0} \frac{\sin 2x}{x}$$

$$\lim_{x \to 0} \frac{2\sin 2x}{2x}$$

$$\lim_{x \to 0} \frac{\tan x}{x}$$

$$\lim_{x \to 0} \frac{2x + \sin x}{4x}$$

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Numbers 7, 11, 19, 21, 24, 25, 26,37, 38, 39, 40, 41, 49,51,54, 65 - 70

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